CASSON NANOFLUID FLOW OVER A VERTICAL PLATE AND ITS THERMOPHYSICAL PROPERTIES

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Abstract

This paper considered the problem of Casson nanofluid flow over a vertical plate with variable thermal conductivity and the effects of its thermo-physical properties. The governing partial differential equations were reduced to nonlinear coupled ordinary differential equations using similarity variables. The coupled nonlinear ordinary differential equations were solved numerically using shooting method with Runge-Kutta fourth order scheme and simulated using MATLAB. The obtained result showed that increasing magnetic field and Casson fluid led to a reduction in the skin friction coefficient, Nusselt number and Sherwood number. Similarly, as Prandtl number becomes large, the local skin friction coefficient, Nusselt and Sherwood numbers reduces respectively.

Keywords: Casson fluid parameter, Brownian motion, MHD, Thermophysical parameter, Thermal diffusivity.

Introduction

The dynamics of electrically conducting fluids; nanoparticles and base fluids under laboratory conditions, shows that the result are nanofluid in nature. Nanofluid combines base fluids such as water or oil and nanoparticles such as Silver(Ag), Copper (II) oxide (CuO) and Aluminum Oxide (Al_2O_3) to improve heat transfer processes. This fluids is applicable in vehicle thermal management, fuel cells, microelectronics, hybrid powered engines, pharmaceutical processes, engine cooling, heat exchanger, and nuclear reactor coolant. Other industrial uses are in transportation industries, energy supply, electronics, textiles, and paper industries (Gbadeyan *et al.*, 2020;Makinde, 2022).

The application of nanofluids has become one of the most researched fluid in the world today by many researchers and as a result of its thermal enhancement, performance, applications and benefits in several important areas, in the field of manufacturing, advanced nuclear systems, transportation, etc .Elbashbeshy and Ibrahim (1993) analyzed the steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. Wasp (1997) analyzed the solid-liquid flow slurry pipeline transportation. Pak and Cho (1998) studied hydrodynamic and heat transfer of dispersed fluids with submicron metallic oxide particles and experimental heat transfer. Xuan and Roetzel (2000) discussed the Conceptions for heat transfer correlation of nanofluids. Maiga *et al.* (2004) discussed heat transfer behaviours of nanofluids in a uniformly heated tube. Seddeek and Abdelmeguid (2006) carried out the effects of radiation and thermal diffusivity on heat transfer over stretching surface with variable heat flux. Oyem (2015) studied effects of thermo-physical properties on free convective heat and mass transfer flow over a vertical plate. Widodo *et al.* (2017) looked into the effect of mhd nanofluid flow through porous cylinder. Swarnalathamma (2018) researched the heat and mass transfer on mhd flow of nanofluid with thermal slip effects. Babu *et al.*(2018) studied the effect of variable viscosity and thermal conductivity on heat and mass transfer flow of nanofluid over a vertical cone with chemical reaction. Fenuga *et al.* (2018) analyzed thermal boundary layer flow over a vertical plate with electrical conductivity and convective surface boundary conditions. Gbadeyan *et al.* (2019) discussed effect of variable thermal conductivity and viscosity on casson nanofluid flow with convective heating and velocity slip. Naduvinamani and Shankar (2019) analyzed heat and mass transfer in squeezing flow of casson fluid with mhd effect. Gopal and Kishan (2019) analyzed Brownian motion and thermophoresis effects on casson nanofluid over a chemically reacting stretching sheet with inclined magnetic field. Irfan *et al.* (2019) studied mhd free stream and heat transfer of nanofluid flow over an exponentially radiating stretching sheet with variable fluid properties. Ismail *et al.* (2019) investigated the effect of variable thermal conductivity on the mhd boundary layer of casson nanofluid over a moving plate with variable thickness. Ali *et al.*(2019) analyzed nanofluid heat and mass transfer in a deformable and peristaltic pump. Ismail *et al.*(2019) investigated the effect of variable thermal conductivity on the mhd boundary layer of casson-nanofluid over a moving plate with variable thickness. Ahmad *et al.* (2019) studied heat transfer analysis for casson fluid flow over stretching sheet with newtonian heating and viscous dissipation. Odesola *et al.* (2020) carried out the analysis of mhd nanofluid flow with heat and mass transfer over a porous stretching sheet. Govardhan *et al.* (2020) investigated the heat and mass transfer in mhd nanofluid over a stretching surface along with viscous dissipation effect. Idowu *et al.* (2020) studied mhd free convective heat and mass transfer flow of dissipating casson fluid with variable viscosity and thermal conductivity effect. Abbas and Magdy(2020) studied the heat and mass transfer analysis of nanofluid flow based on Copper(Cu), Aluminum oxide (Al_2O_3)and Titanium (II) oxide TiO_2 over a moving rotating plate. Safiei *et al.*(2020) reviewed the thermal conductivity and dynamic viscosity of nanofluids. Muhammad *et al.* (2020) analyzed the entropy generation in mhd casson fluid flow with variable heat conductance and thermal conductivity over non linear bidirectional stretching surface. Kigio *et al.*(2021) investigated analysis of volume fraction and convective heat transfer on mhd casson nanofluid over a vertical plate. Jawad *et al.*(2021) studied the analytical of mhd mixed convection flow for Maxwell nanofluid with variable thermal conductivity and soret and dufour effects. Chandrasekhar et al. (2022) Studied casson-nano-magneto hydrodynamics boundary layer fluid flow towards a stretching sheet including the effects of cross diffusion, velocity and thermal wall slips. Shamshuddin et al. (2022) discussed the thermophoretic movement transport of reactive Casson nanofluid on riga plate surface with nonlinear thermal radiation and uneven heat sink/source. Roy et al. (2022) studied the influence of activation energy and nonlinear thermal Radiation with ohmic dissipation on heat and mass transfer of a Casson nanofluid over stretching sheet.

From the above literature, the concept of variability on thermal conductivity and its effects on casson nanofluids have not been exhausted, hence, this study. This paper therefore, extends the work of Kigio *et al.* (2021) to include variable thermal conductivity over a vertical plate of an electrically conducting mhd casson nanofluid flow over a vertical plate to obtain its thermo-physical properties and their effects on velocity, temperature, and concentration profiles.

Mathematical Formulation

The equations governing the convective flow of an electrically conducting Casson nanofluid over a vertical plate in the presence of variable thermal conductivity and magnetic field are given in equations (1) - (4) with boundary conditions (5) - (6). A magnetic field (B_o) is applied normal to the flow and the flow in the x −axis is taken along the vertical plate in the upward direction and y −axis is normal to the plate (Elbashbeshy and Ibrahim, 1993; Kigio *et al*. 2021) see Fig. 1.

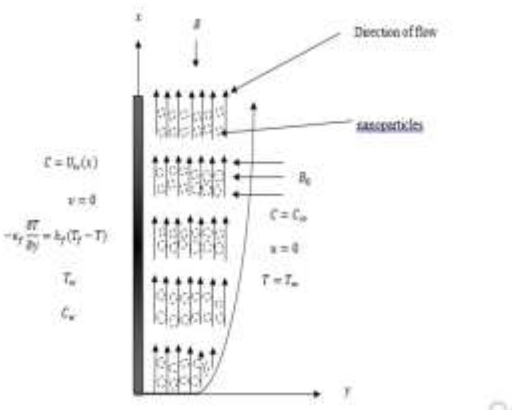

Figure 1: Schematic Diagram

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty)-\frac{\sigma_{nf}B_0^2u}{\rho_{nf}} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\kappa(T)\frac{\partial^2 T}{\partial y^2}+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)+\frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)+\frac{\sigma_{nf}B_0^2u^2}{(\rho c_p)_{nf}} \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

subject to the boundary conditions:

$$u=ax \quad v=0 \quad -k_f\frac{\partial T}{\partial y}=h_f(T_f-T) \quad C=C_w(x) \quad at \quad y=0 \qquad (5)$$

$$u\to 0 \quad T\to T_\infty \quad C\to C_\infty \quad as \quad y\to\infty. \qquad (6)$$

WhereT_w,C_w,T_∞ and$C_\infty$are the wall temperature, concentration free stream temperature and concentration respectively,u, v are velocity components along thex, y -axis of the flow, ρ is density of the fluid, c_p is the specific capacity at constant pressure, g is the acceleration due to gravity, T is the temperature of the fluid, C is the dimensional concentration in the fluid.The following conditions are proposed and assumed such that the effective density, viscosity, specific heat capacity, and thermal conductivity of the nanofluid are given in equations (7) - (10) (Wasp, 1997; Pak and Cho, 1998; Xuan and Roetzel, 2000; Oyem,2015; Kigio *et al*. 2021)

$$\frac{k_{nf}}{k_{bf}}=\frac{k_{nf}+2k_{np}-2\phi\left(k_{bf-k_{np}}\right)}{k_{np}+2k_{bf}+2\phi(k_{bf}-k_{np})} \qquad (7)$$

$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_{bf}+\phi(\rho c_p)_{np}, \qquad (8)$$

$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_{bf}+\phi(\rho c_p)_{np} \qquad (9)$$

$$\frac{\mu_{nf}}{\mu_{nf}v_{bf}}=\frac{0.904e^{0.148\phi}}{\left(1-\phi+\phi\frac{\rho_{np}}{\rho_{bf}}\right)} \qquad (10)$$

In this study, the fluid thermal conductivity $\kappa(T)$ is assumed to vary linearly in the form(,Elbashbeshy, and Ibrahim, 1993;Seddeek, and Abdelmeguid,2006; Oyem, 2015)

$$\kappa(T)=k^*(1+\gamma\theta) \qquad (11)$$

where,$\gamma=\delta(T_w+T_\infty)$ is the thermal conductivity variation parameter of the flow and k^* is the ambient fluid thermal conductivity.Consequently, in order to resolve the governing partial differential equations (1)–(4) subject to the boundary conditions (5)–(6), the following similarity variables are considered in equation (12)–(15) below

$$n=y\sqrt{\left(\frac{a}{v}\right)}\ ,\ u=\frac{\partial\psi}{\partial y};\quad v=-\frac{\partial\psi}{\partial x}\ ;\quad \psi=\sqrt{avx}f(\eta) \qquad (12)$$

$$T=T_\infty+(T_w-T_\infty)\theta(\eta) \qquad (13)$$

$$C=C_\infty+(C_w-C_\infty)\phi(\eta) \qquad (14)$$

$$u=axf',\ \ v=-\sqrt{av}f(\eta) \qquad (15)$$

Applying equations(12)–(15) on equations(1)–(6); equation (1) is satisfied and equations(2)–(4)with conditions(5)–(6) reduces to;

$$\frac{\mu_{nf}}{\rho_{nf v_{bf}}}\left(1+\frac{1}{\gamma}\right)f'''-(f')^2+ff''+Gr_t\theta+Gr_s\phi-Mf'=0 \qquad (16)$$

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$$B_1k^*(1+\gamma\theta)\theta'' + B_1k^*\gamma(\theta')^2 + P_rf\theta' + N_b\phi'\theta' + N_t(\theta')^2 + \frac{\mu_{nf}}{\rho_{nf}v_{bf}}P_rE_C(f'')^2 + \ldots + P_rME_C(f')^2 \quad (17)$$

$$\phi'' + S_c\phi' f + \frac{N_t}{N_b}\theta'' = 0 \quad (18)$$

With the dimensionless conditions as;

$$f(0)=0,\ f'(0)=1,\ \theta'(0) = -B_i(1-\theta),\ \phi(\eta) = 0 \quad (19)$$

$$f' \to 0;\ \theta \to 0;\ \phi \to 1 \quad (20)$$

From equations (16) -(18),

$$Gr_t = \frac{g\beta(T_w - T_\infty)}{a^2x};\ Gr_s = \frac{g\beta^*(C_w - C_\infty)}{a^2x};\ S = \frac{v}{D_B};\ Ec = \frac{a^2x}{(c_p)_{nf}(T_w - T_\infty)};\ Pr = \frac{v_{bf}}{\alpha_{bf}};$$

$$M = \frac{\sigma_{nf}B_o^2}{\alpha\rho_{nf}};\ N_b = \frac{\tau D_B}{\alpha_{nf}};\ N_t = \frac{\tau D_T(T_w - T_\infty)}{\alpha_{nf}T_\infty};\ Bi = \frac{h_f}{k_f}\sqrt{\left(\frac{a}{v}\right)};$$

$$B_1 = \frac{\left(\frac{k_{nf}+2k_{np}-2\phi(k_{bf}-k_{np})}{k_{np}+2k_{bf}+2\phi(k_{bf}-k_{np})}\right)}{\left((1-\phi)+\phi\frac{(\rho c_p)_{np}}{(\rho c_p)_{bf}}\right)}$$

such that f, θ,ϕ and η are the field velocity, temperature concentration and dimensionless boundary layer coordinate of the fluid respectively. In order to solve the coupled nonlinear ordinary differential equations (16) – (18), the equations are further reduced to a system of first order linear equations. Set,

$$X_1 = f;\ X_2 = f';\ X_3 = f'';\ X_4 = \theta;\ X_5 = \theta';\ X_6 = \phi;\ X_7 = \phi' \quad (21)$$

such that,

$$X_1' = X_2 \quad (22)$$

$$X_2' = X_3 \quad (23)$$

$$X_3' = \frac{X_2{}^2 - X_1X_3 - Gr_tX_4 - Gr_sX_6 + MX_2}{\frac{\mu_{nf}}{\rho_{nf}v_{bf}}\left(1+\frac{1}{\gamma}\right)} \quad (24)$$

$$X_4' = X_5 \quad (25)$$

$$X_4' = -\frac{P_rX_1X_5}{A_1k^*(1+\gamma\theta)} - \frac{k^*\gamma X_5{}^2}{A_1k^*(1+\gamma\theta)} - \frac{N_bX_5X_7}{A_1k^*(1+\gamma\theta)} - \frac{N_tX_5{}^2}{A_1k^*(1+\gamma\theta)} - \frac{\mu_{nf}}{\rho_{nf}v_{bf}A_1k^*(1+\gamma\theta)}P_rE_C\,X_3{}^2 - \frac{P_rME_C\,X_2{}^2}{A_1k^*(1+\gamma\theta)} \quad (26)$$

$$X_6' = X_7 \quad (27)$$

$$X_6' = -S_CX_1X_7 - \frac{N_t}{N_b}X_6' \quad (28)$$

with initial conditions

$$X_1(0)=0,\ X_2(0)=1,\ X_3(0)=s_1,\ X_4(0)=0,\ X_5(0)=s_2, X_6(0)=0,\ X_7(0)=s_3.$$

Where,$s_1 = f''$; $s_{2,} = \theta'$; $s_3 = \phi'$ such that the values ofs_1,$s_{2,}$ and s_3 are obtain using the shooting method to ensure that$X_5(0) = B_i(X_4(0)-1)$, $X_2(\infty) \to 0$ $X_4(\infty) \to 1$ are satisfied.

Results and Discussion

Mhd casson nanofluid flow over a vertical plate with variable thermal conductivity in the presence of magnetic field have been considered. The governing partial differential equations (1) – (4) subject to the boundary conditions (5)–(6) were reduced using similarity variables (12)–(14) along with base and nanoparticles in (7) – (10) reduced to a coupled nonlinear ordinary differential equations (16) – (18) with dimensionless condition in (19) – (20). Equations (16) – (20) were further reduced to a system of first order ODE and solve numerically by shooting method with Runge-Kutta fourth order scheme using MATHLAB. The effects of the thermo-physical properties on velocity, temperature, and concentration profiles are presented in tables and displayed in graphs. The choice of parameters were in agreement with (Kigio *et al*. 2021) such that;

$\gamma = 1$; $Gr_t = 1$; $Gr_s = 3$; $M = 3$; $P_r = 7.62$; $E_c = 0.1$; $\psi = 0.01$;
$N_t = 0.1$; $N_b = 0.1$; $S_c = 0.62$; $B_i = 0.1$; $\kappa = 0.7$; $\alpha = 0.5$;

and the thermo-physical properties of the nanofluid is shown in Table 1.

Table 1: Thermo-physical properties of Nanoparticles

Materials	ρ	c_p	κ
Engine oil	804	1909	0.145
Cu	8933	385	401

The effects of magnetic force field with velocity profiles are shown in Figs. (1)–(3). In Fig. 1, as magnetic field increases, the flow profiles decreases along the boundary layer toward the free stream thereby, inducing the Lorentz force. The Lorentz force, opposes the flow field by reducing the flow velocity. This result is in agreement with Chen *et al*. (1998), Ahmad *et al*. (2018), Farooq *et al*.(2019),Bulinda *et al*. (2020) and Kigio *et al*. (2021). In Figs. (2) and (3), increase the effect of magnetic field, results to increase away from the plate in both temperature and concentration profiles to the free stream. The internal friction generated from the reduced velocity produces more heat energy thereby increasing temperature and concentration of the flow and this agrees with Chen *et al*. (1998), Ahmad *et al*. (2018), Farooq *et al*. (2019), Bulinda *et al*. (2020) and Kigio *et al*. (2021). Figs. (4) to (6) shows the effects of Prandtl number on velocity, temperature, and concentration profiles. Increase in Prandtl number consequently means an increase in momentum diffusivity and a decrease in the thermal diffusivity. The viscous boundary layer thickness increases with increase in the momentum diffusivity and hence reduces the flow velocity. Fig. (4) show that the velocity, thermal boundary layer thickness and concentration profiles reduces as Prandtl increase number increases along the plate toward the free stream. Due to thermal enhancement in temperature, the material particles (casson fluid) needed extra time for heat transfer for its surrounding particles. However, increase inPr brings down energy,concentration and velocity profiles. This is in agreement with Idowu*et al*. (2020). The effects of thermophoretic parameters on the velocity, temperature and concentration profiles are displayed in Figs. (7) to (9). The thermophoretic parameter (N_t) 5 represents the rate at which the haphazard motion among the nanoparticles takes place within the flow. From figures (7) and (8), increase in N_t, reduces the nanoparticles flow in the velocity and concentration profiles as a result of the drags produced by the nanoparticles. However, the the temperature profiles increases rapidly away from the plate and later exhibited the haphazard motion and reduces gently toward the free stream due to the energy produced by the rapid moving particles. The effects of Brownian motion (N_b) on the thermal profiles are shown in Figs. (10) to (12). Increase in N_b parameter, increases the flow velocity, temperature and concentration profiles. This is because; increasing Brownian motion increases the internal energy of the system. The variation of casson fluid parameter (γ) decreases both in velocity and temperature profiles as shown in Figs. (13) and (14). The large and small value of γ parameter corresponds to Newtonian and non-Newtonian fluids thus,γ reduces the yield stress. This result corresponds with Idowu *et al*. (2020). Increasing the thermal diffusivity parameter α reduces the temperature profiles at the boundary layer but increases the temperature profile at the free stream. As shown in Fig (15).

The thermo-physical interaction of the Skin-friction, Nusselt and Sherwood numbers with magnectic field M and casson fluid parameter γ are displayed in figure (16) and (17). It is observed from figure (16) and (17) that increase in $M$and γ, reduces the Skin-friction negatively away from the free stream, but shows a stable flow interaction with the Nusselt and Sherwood but reduces gently away from the plate towards the free stream. The relations between the Prandtl number and the Skin-friction coefficient, Nusselt and Sherwood number are shown in figure (18) and tabled (2). The analysis shows that as P_r becomes large, C_f, N_u and S_h reduces.

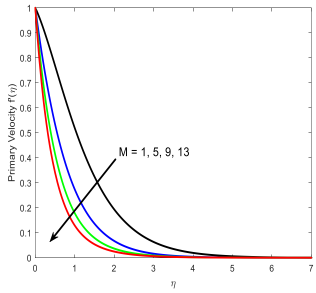

Fig. 1. Effects of M on velocity Profiles

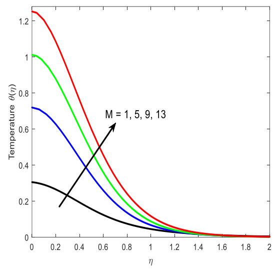

Fig. 2. Effects of M on temperature profiles

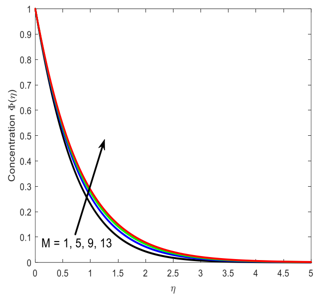

Fig. 3. Effects of M on concentration profiles

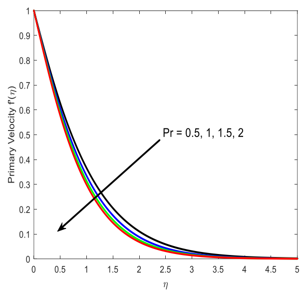

Fig. 4. Effects of P_r velocity profiles

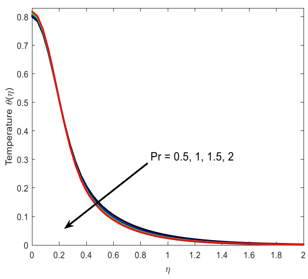

Fig. 5. Variation of P_r on temperature profiles

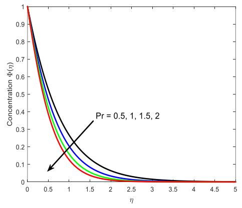

Fig. 6. Effects of P_r on concentration profiles

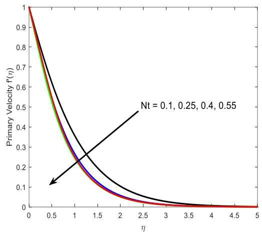

Fig. 7. Effects of N_t on velocity profiles

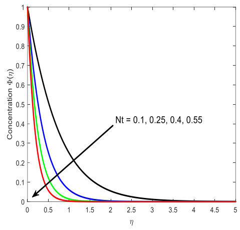

Fig. 8. Effects of N_t on temperature profiles

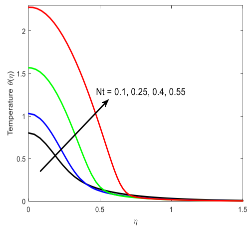

Fig. 9. Effects of N_t on concentration profiles

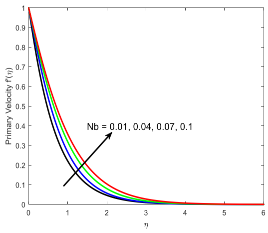

Fig. 10. Variation of N_b against velocity profiles

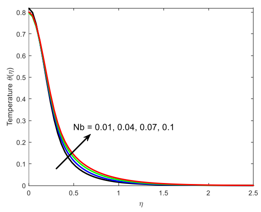

Fig. 11. Variation of N_b against temperature profiles

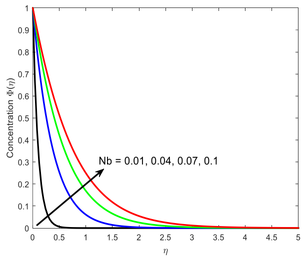

Fig. 12. Variation of N_b against concentration profiles

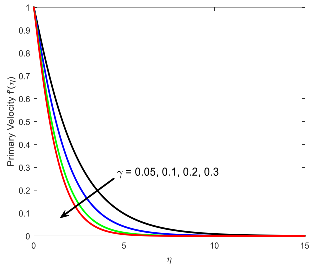

Fig. 13. Variation of velocity with Casson fluid parameter

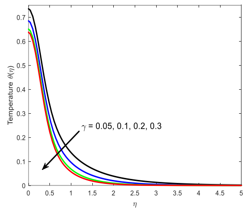

Fig. 14. Variation of temperature with Casson fluid parameter

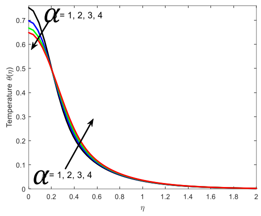

Fig. 15. Variation of temperature with variable thermal conductivity

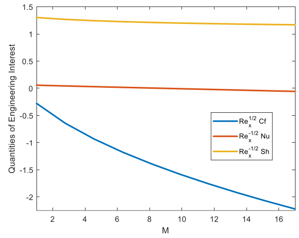

Fig. 16. Variation of quantities of interest with magnetic

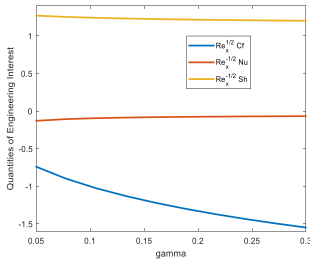

Fig. 17. Variation of quantities of interest with Casson fluid parameter

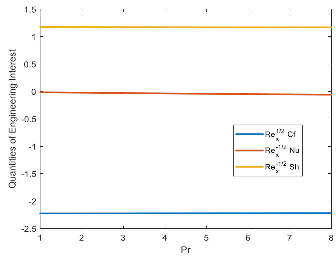

Fig. 18. Variation of quantities of interest with Prandtl number

Table 2. Variation of quantities of interest with Prandtl number

P_r	1.0000	1.7778	2.5556	3.3333	4.1111	4.8889	5.6667	6.4444	7.2222	8.0000
$Re^{1/2}Cf$	-0.2770	-0.6433	-0.9320	-1.1759	-1.3901	-1.5830	-1.7596	-1.9233	-2.0764	-2.2208
$Re^{1/2}Nu$	-0.0156	-0.0210	-0.0267	-0.0321	-0.0373	-0.0421	-0.0467	-0.0511	-0.0553	-0.0593
$Re^{1/2}Sh$	1.1762	1.1742	1.1728	1.1718	1.1710	1.1704	1.1695	1.1695	1.1692	1.1689

Conclusion

A magnetohydrodynamic heat and mass transfer of a Cassonnanofluid flow over a vertical plate in the presence of magnetic field and variable thermal conductivity have been studied. From the study, the following were observed that;

i. Magnetic field decreases with velocity profile but increases in temperature and concentration profiles.
ii. Increasing the interaction of thermal diffusivity parameter reduces the temperature profile along the plate but later increases away from the plate to the stream.
iii. As magnetic field and Prandtl number becomes large, the local skin friction, Nusselt number and Sherwood number reduces in a stable form away from the plate.

NUMENCLATURE

B_i - Biot number
B_o - Magnetic field strength
C - Concentration of nanoparticle
c_p - Specific heat capacity
C_w - Wall stream concentration
C_∞ - Free stream concentration
D_B - The Brownian diffusion coefficient
D_T - Thermophoretic diffusion coefficient
E_c - Eckert number (or viscous dissipation parameter)
g - Accelerated due to gravity
Gr_t - Thermal Grashof parameter
Gr_s -SolutalGrashof parameter
k-Thermal conductivity
k^* -Thermal conductivity of the ambient fluid
k_{nf} - Thermal conductivity of nanofluid
k_{bf} -Thermal conductivity of basefluid
k_{np} -Thermal conductivity of nanoparticle
M -Magnetic field parameter
n -Velocity index
N_b -Brownian parameter
N_t - Thermophoretic parameter
P_r - Prandtl number
S_c - Schmidt number
T - Temperature of the fluid within the boundary layer
T_w - Wall stream temperature
T_∞ - free stream temperature
u - Velocity component in x– direction
v - Velocity component in y– direction
x - Coordinate along the plate
y - Coordinate normal to the plate

Greek Letters

α - Thermal diffusivity
α_{bf} - Thermal diffusivity of base fluid
α_{nf} - Thermal diffusivity of the nanofluid
β_T- - Coefficient of thermal expansion
β_C - Coefficient of concentration expansion
β - Thermal coefficient of volumetric expansion
β^* - Concentration coefficient
$(\rho c_p)_{np}$ - Heat capacity of the nanofluid
δ - Modified Frank Kamenetskii parameter
η - Similarity variable
ψ - Stream function
∞ - Evaluation at free stream condions
ϕ - Dimensionless concentration
ρ - Density of fluid
ρ_{nf} - Density of the Nanofluid
σ - Electrical conductivity
υ - Kinematic viscosity
μ- Coefficient of dynamic fluid viscosity
μ_{nf} - Dynamic viscosity of the Nnofluid
γ - Casson fluid parameter or Thermal conductivity variation parameter\\
τ - Shear stress

Subscripts

bf -Base fluid
nf - Nanofluid
np – Nanoparticle

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Competing Interests

Authors have declared that no competing interests exist

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